

A COMPARATIVE OF CURVE-FITTING ALGORITHMS FOR THE EXTRACTION OF MODAL PARAMETERS FROM RESPONSE MEASUREMENTS

UN ESTUDIO COMPARATIVO DE ALGORITMOS DE AJUSTES DE CURVA PARA LA EXTRACCIÓN DE PARÁMETROS MODALES DE MEDICIONES DE VIBRACIONES

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ABSTRACT

The main objective of this paper is to perform a comparison of several curve-fitting methods for extraction of the modal parameters from response vibration measurements, and in particular the best damping estimates. Measurements were carried out on a steel beam to which a constrained layer had been added to make the damping more similar to that of vehicle structural components. Two shakers with different excitation signals, a periodic impulsive and a random signal, respectively, excited the structure, but after separation, only the random part was analysed for the results of this paper. This study compares a number of common curve fitting methods, viz: The Rational Fraction Polynomial Method, the Complex Exponential Method, the Complex Cepstrum Method, the Hilbert Envelope Method and the Ibrahim Time Domain method. The most accurate results for detection of the damping and natural frequencies were obtained by using the Ibrahim Time Domain Method, with the Rational Fraction Polynomial method very similar. The Hilbert Envelope method gave comparable damping estimates. The Cepstrum and Complex Exponential methods gave reasonable results for the frequencies, but not for the damping.

RESUMEN

El objetivo principal de este trabajo es realizar una comparación de varios métodos de ajuste de curvas para la extracción de los parámetros modales a partir de mediciones de vibración de respuesta y, en particular, definir las mejores estimaciones de amortiguación. Las mediciones se llevaron a cabo en una viga de acero y se adhirió una capa de material de amortiguación, esto para hacer que la amortiguación experimental sea similar a la de los componentes estructurales en los vehículos. Dos agitadores "shakers" con diferentes señales de excitación se aplicaron, una impulsiva periódica y una señal aleatoria, respectivamente. En este trabajo de investigación, después de la separación de las señales; sólo la señal aleatoria se utilizó para futuros análisis. Este estudio se realizó la comparación de métodos más comunes de ajuste de curvas: El método de la fracción racional de polinomio, el método de exponencial compleja, el Método Cepstrum Complejo, el método de Hilbert Envoluta y el método Ibrahim dominio de tiempo.

Los resultados más precisos obtenidos para la detección de las frecuencias naturales y de amortiguación fue con el método de Ibrahim en el dominio de tiempo, y con resultados similares con el método de fracción racional de polinomio. El método de Hilbert Envoluta dio estimaciones de amortiguación comparables. El método de Cepstrum y el Método Exponencial Compleja dieron resultados comparativos para las frecuencias pero no para los valores de amortiguación.

Keywords: Rational Fraction Polynomial, Complex Exponential, Complex Cepstrum, Hilbert Envelope, Ibrahim Time Domain Method.

Palabras Clave: Fracción Racional de Polinomio, Exponencial Compleja, Cepstrum Complejo, Hilbert Envoluta y Método de Ibrahim en el Dominio de Tiempo.

1. INTRODUCTION

Due to the large amount of literature and algorithms currently available for curve fitting structural data, it has become difficult to determine the optimum method for each situation. Therefore, the main objective of this paper is to perform a comparison of several curve-fitting methods for extraction of the modal parameters from response vibration measurements, and in particular the best damping estimates. To define the poles, including frequency and damping, of a system, from the response vibration responses, has always been a difficult task.

Damping exists in all vibratory systems whenever there is energy dissipation. For assembled metal structures most damping is at joints or in attached damping materials. For free vibration, the loss of energy from damping in the system

results in the decay of the amplitude of motion. In forced vibration, loss of energy is balanced by the energy supplied by the excitation. In either situation, the effect of damping is to remove energy from the system. In mathematical formulations the damping force is usually treated as viscous, and assumed proportional to velocity. However, this does not mean that the physical damping mechanism is viscous in nature; it is simply a convenient modelling method.

In this work, measurements were carried out on a steel beam to which a constrained layer had been added to make the degree of damping more similar to that of vehicle structural components. Two shakers with different excitation signals, a periodic impulsive and a random signal, respectively, excited the structure. It had been demonstrated in previous research [1] that it was possible to consider each component separately, and gain information from each that together provides a better estimate of the system response. With a single periodic impulsive excitation, the periodic response could be used to determine the residues, or equivalently the zeros of the system Frequency Response Function (FRFs), but the lack of resolution gave poor damping estimates. More accurate pole frequency and damping estimates could be obtained from the random part, even where this resulted from multiple or distributed sources. The current comparison of analysis methods applies to the random part only. Where necessary, the system was considered to be minimum phase, meaning that only the log amplitude spectra were required. The following common curve fitting methods were used for the comparison: the Rational Fractional Polynomial method (RFPM), the Complex Exponential method (CEM), the Hilbert Envelope method (HEM), the Cepstrum method (CM) and the Ibrahim Time Domain method (ITDM). The flowchart of Fig. 1 shows the process used to identify the poles (including damping).

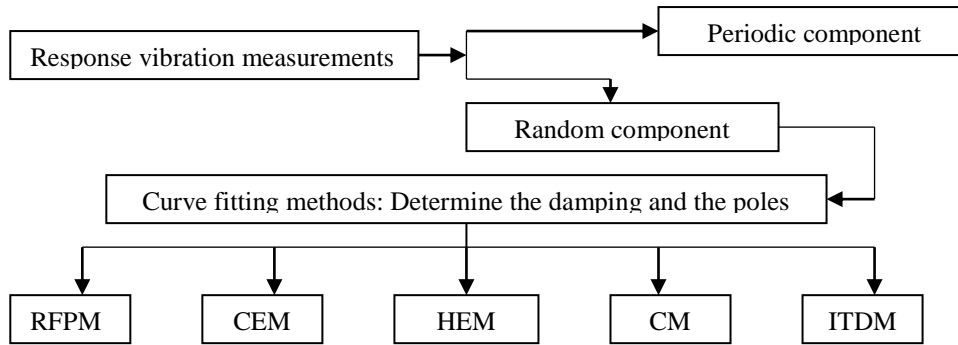


Figure 1 - The flowchart of the analysis procedure.

2. THEORETICAL BACKGROUND

a) The Cepstrum Method (CM)

Cepstrum analysis was first defined as far back as 1963 by Bogert *et al* [2]. It was proposed at that time as a better alternative to the autocorrelation function for the detection of echoes in seismic signals. The word *cepstrum* was coined by reversing the first syllable in the word *spectrum*. The cepstrum exists in a domain referred to as *quefrequency* (reversal of the first syllable in *frequency*), which has units of time. In later years this method has had a variety of applications in areas such as speech and image processing, and gear and reciprocating machine diagnostics. Generally the cepstrum is defined as the inverse Fourier transform of the logarithmic spectrum [3].

$$C(\tau) = F^{-1}[\log(X(k))] \quad (1)$$

where C is the complex cepstrum.

The *real* cepstrum of a digital signal $x(n)$ is defined as:

$$C(n) = idft(\ln|X(k)|) \quad (2)$$

with $X(k)$ as the discrete Fourier transform (dft) of $x(n)$ and idft as the inverse discrete Fourier transform.

The *complex cepstrum* is defined as:

$$\hat{x}(n) = idf(\ln|X(f)| + j \arg(X(k))) \quad (3)$$

which is actually real since the log amplitude is even and the phase is odd. Note that the complex cepstrum of a minimum phase function is causal and can be obtained from the real cepstrum by setting negative quefrency components to zero, and doubling positive quefrency components, since the phase of the spectrum is the Hilbert transform of the log amplitude [4]. Thus, in that case the phase does not have to be measured or unwrapped. This assumption was made in this paper, and used also to generate complex spectra from log amplitude spectra. To curve fit the cepstrum data and obtain the poles and zeros, the software developed earlier [5] was used.

b) The Complex Exponential Method (CEM)

The well-known curve-fitting algorithm called the Complex Exponential Method (CEM), or Prony Algorithm works in the time domain. The time series are represented as sums of complex exponential functions [6] representing impulse responses.

$$\hat{x}(n) = \sum_{k=1}^P h_k z_k^{n-1} \tag{4}$$

where

$$h_k = A_k e^{j\Phi_k} \tag{5}$$

and

$$z_k = e^{-\alpha_k T_e} \cdot e^{j2\pi f_k T_e} \tag{6}$$

Here, A_k is the amplitude of the complex exponential, α_k is the damping factor, Φ_k is the initial phase, T_e is the sample interval, and f_k is the natural frequency. The CEM algorithms are multi-step procedures: An AR (Autoregressive) fit is first performed, the roots are found corresponding to the AR coefficients for the complex exponential (frequency and damping) parameters. The roots are then filtered and least squares fitted for the complex amplitude parameters. The complex exponentials give the corresponding damping factors and the natural frequencies. The damping can be calculated by the following equation:

$$\alpha_k = -Ln(abs(z_k))/T_e \tag{7}$$

c) The Hilbert Envelope Method (HEM)

The complex exponentials of Equation (6) have an envelope defined by the first part, and as shown in Equation (7), the (negative of the) slope of the logarithm of the envelope defines the damping coefficient. Where the different natural frequency peaks are separated, they can be filtered out in the frequency domain and inverse transformed to give the equivalent single degree of freedom (SDOF) impulse response, from whose envelope the damping can be obtained. Figure 2 shows the impulse response of an SDOF system and how its envelope can be obtained by means of a Hilbert transform. In fact, if a one-sided spectrum is inverse transformed to the time domain, the resulting time signal is analytic, meaning that its imaginary part is automatically the Hilbert transform of the real part and its modulus is the envelope [3]. Since the Hilbert transform corresponds to a convolution with a hyperbolic function, it cannot produce a sudden step as illustrated in Fig. 2(b), but a little away from the initial part of the response the estimated envelope is accurate and its logarithm gives a straight line whose slope is the negative of the damping coefficient.

d) The Ibrahim Time Domain Method (ITDM)

This method was introduced in 1977 [8], and it has been widely used by the aerospace community for identification of modal parameters. Gao [9] presented an extensive research work using the ITD for extraction of modal parameters. The ITD method uses structural free response data to construct two response matrices with a certain time delay between them. The delay relationship is used to form an Eigen-Value Problem (EVP) [9]. The natural frequencies, damping factors and modes can be extracted by solving the EVP. The ITD method is based on structural free response data and can deal with different type of signals, such as from impact excitation, random decrement of random processes or from turning off of any type of continuous excitation. The ITD method allows an oversized mathematical model to provide an outlet for various types of noise contained in the measured response data. The use of an oversized model, however, gives rise to the difficulty of distinguishing true physical from computational modes. To overcome this, Gao put forward a mode-distinguishing factor, the Mode Shape Coherence and Confidence Factor (MSCCF)[9]. It was found that this factor was very powerful in distinguishing true physical modes from noise modes and frequency folded and overlapped modes (where, because of aliasing the true angle of a pole in the z-plane, could be outside the range $[0:\pi]$). The reader is referred to [9] for a more detailed explanation.

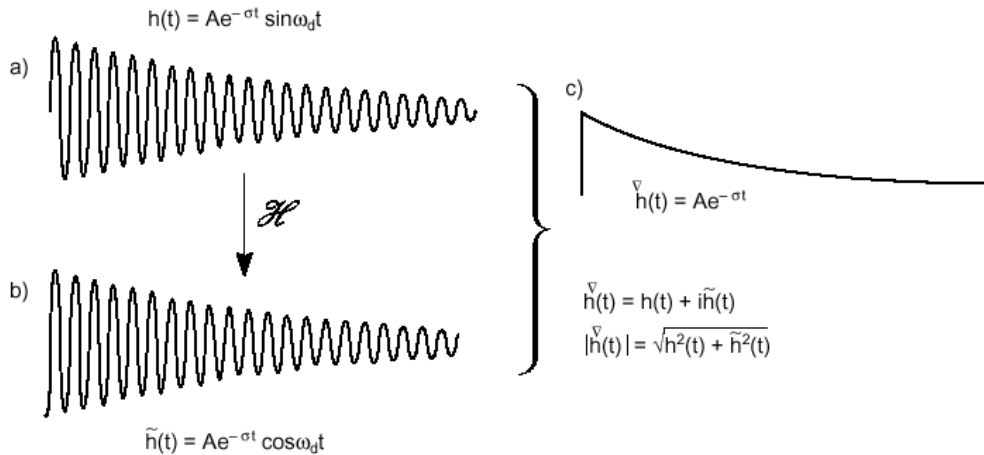


Figure 2 - The Hilbert transform enables computation of the envelope of the Impulse-response function [7].

e) The Rational Fraction Polynomial Method (RFPM)

The rational fraction form of a transfer function is the ratio of two polynomials, with the roots of the numerator giving the zeros, and the roots of the denominator giving the poles (Equation (8)). In general, the orders of the numerator and the denominator polynomials are independent of one another. The denominator is referred to as the characteristic polynomial of the system. By curve fitting FRF (frequency response vibration data) against the analytical form in Equation (8), and then solving for the roots of both the numerator and the characteristic polynomials, the zeros and poles (including damping) of the transfer function can be determined. Curve fitting in the RFPM consists of finding the unknown ($a_k, k=0, \dots, m$) and ($b_k, k=0 \dots n$) such that the error between the analytical expression (8) and the FRF is minimized over the chosen frequency range. To compensate for the effects of unmeasured out-of-band modes, extra zeros are typically added to the model, which helps to correct the residues when the conversion from a pole-zero to a pole-residue model is made [10].

$$H(\omega) = \left. \frac{\sum_{k=0}^m a_k s^k}{\sum_{k=0}^n b_k s^k} \right|_{s=j\omega} \tag{8}$$

3. MEASUREMENT AND ANALYSIS

Measurements were carried out on a steel beam 1275 mm by 75 mm by 10.3 mm. In order to increase the damping to be of the same order as in vehicle structures, a constrained layer was added to the beam, consisting of a brass shim attached by double-sided tape. The beam was supported on soft springs, so its behaviour in bending was effectively free-free. To generate simultaneous periodic impulsive excitation and broad-band random excitation, two shakers were used. To measure the responses several accelerometers were used. Data was recorded using a B&K Portable Pulse System and the post processing was performed in Matlab. The periodic impulse signal was provided via the Pulse System. The pulse shape was Gaussian, with a standard deviation of 0.2 ms to give a frequency range > 1 kHz. The period was set at 0.5s (giving a resolution of 2 Hz) with 500 repetitions. The measurement set-up can be seen in Figure 3.

As reported in [1], the periodic response was extracted by synchronous averaging, and then subtracted from the total signal to give the response to the random excitation. The periodic response was used to estimate the residues of the frequency responses, but because of poor frequency resolution gave poor estimates of the damping, and only reasonable estimates of the natural frequencies. The random response was used to give better estimates of the system poles (frequency and damping) even though in general the random response may result from multiple or distributed inputs. This paper considers only the analysis of the random part of the response.

An important issue with use of any curve fitter is how to compensate the residual effects of out-of band modes since the measurements are always made over a limited frequency range. As a consequence, the measurements will typically contain the residual effects of resonance, which lie outside the frequency range of the curve-fitting. Regardless of whether a curve fitting method uses the time or frequency domain, the residual effects of out-of-band modes must be dealt with.

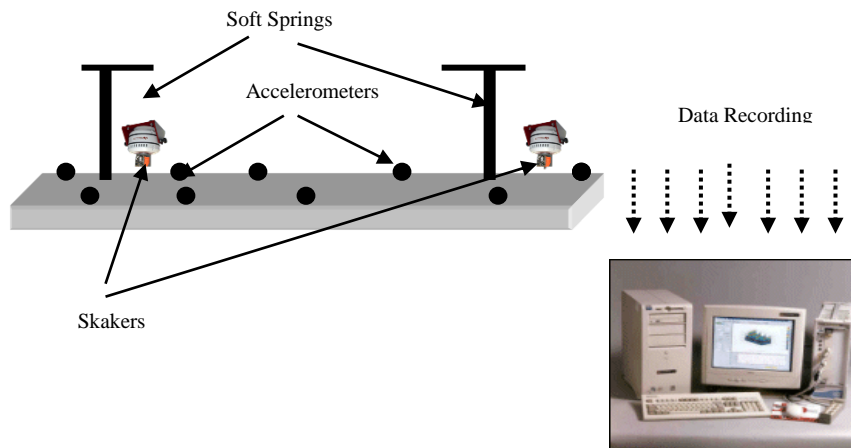


Figure 3 - Measurement set-up.

RFBM offered a unique advantage that was not available with the CEM, for instance. With the RFBM, the out-of-band effects can be approximated by specifying additional terms for either the numerator or the denominator [10]. The Complex Exponential method almost always requires the use of extra modes in order to obtain valid results. The difficulty with using extra modes in the model is that the results must then separate out the “real” modes from the computational modes. The complex cepstrum uses an equalizer function (from a similar case or finite element model) in order to compensate for the out-of-band modes [5]. The Ibrahim time domain method uses the Mode Shape Coherence and Confidence Factor to distinguish the physical modes from computational modes [9]. This study used both simulated data and the experimental data. The simulated data can be seen in Figures 4 and 5. Figure 4 shows: a) The impulse response function, b) The log magnitudes of the mobilities of the individual modes, and c) The log magnitude of the overall mobility. Within the excitation frequency range of 0 Hz to 1024 Hz, six resonance frequencies are clearly seen. Figure 5 compares typical regenerated data with original analytical data.

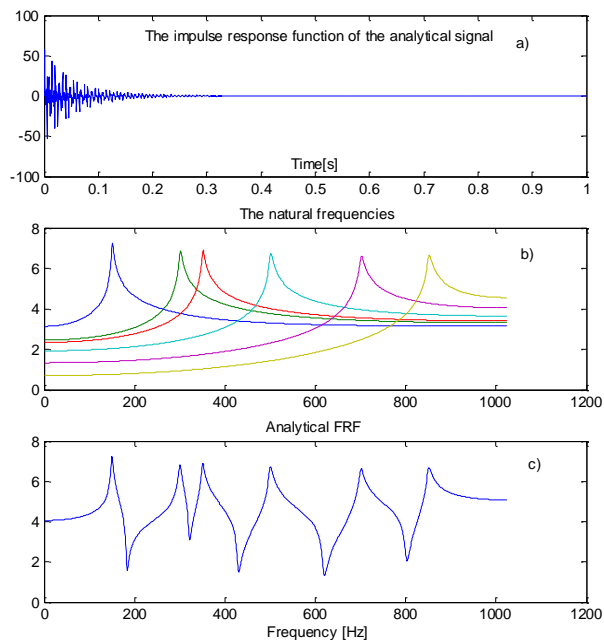


Figure 4 - The analytical signal: a) The impulse response function; b) The individual log magnitudes of mobilities; c) The log magnitude of the overall mobility.

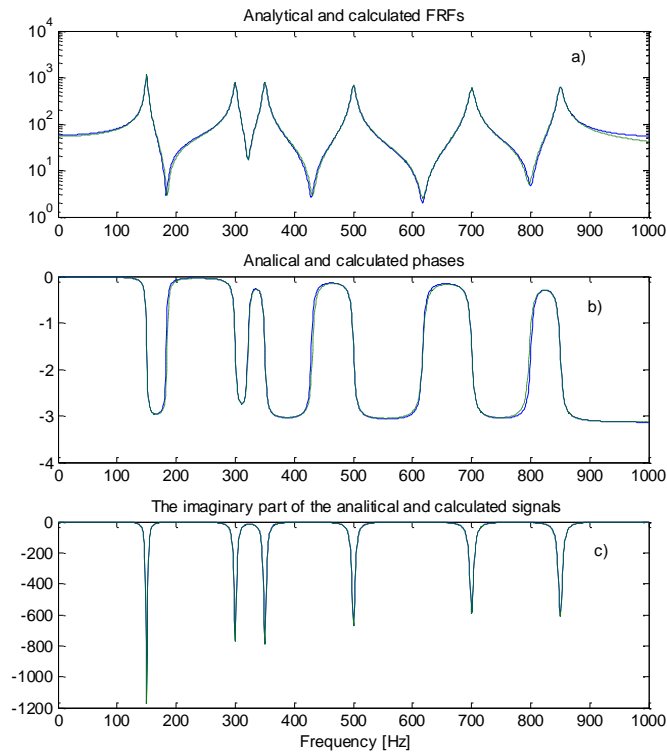


Figure 5 - The analytical and calculated signals: a) The log magnitudes of mobility; b) Analytical and calculated phases; c) Analytical and calculated Imaginary parts

4. RESULTS AND DISCUSSION

The results of the analysis of the simulated signals by the different methods can be seen in Table 1 and Table 2, and compared with the exact values for the simulated data.

TABLE 1 - SUMMARISING THE RESULTS OF THE ESTIMATED FREQUENCIES AND THEIR CALCULATED ERRORS

Freq. [Hz]	CM		CEM		ITM		RFPM	
	Freq. [Hz]	Error [%]	Freq. [Hz]	Error [%]	Freq. [Hz]	Error [%]	Freq. [Hz]	Error [%]
150	148.271	1.153	151.610	1.073	150.07	0.047	150.070	0.047
300	300.182	0.061	297.480	0.840	300.14	0.047	300.140	0.047
350	346.657	0.955	352.120	0.606	350.16	0.046	350.170	0.049
500	502.168	0.434	496.120	0.776	500.24	0.048	500.240	0.048
700	699.925	0.011	702.580	0.369	700.36	0.051	700.340	0.049
850	848.930	0.126	853.280	0.386	850.47	0.055	850.450	0.053

TABLE 2: SUMMARISING THE RESULTS OF THE ESTIMATED DAMPING AND THEIR CALCULATED ERRORS

Damp.* [%]	CM		CEM		ITM		RFPM		HEM	
	Damp. [%]	Error [%]	Damp. [%]	Error [%]	Damp. [%]	Error [%]	Damp. [%]	Error [%]	Damp. [%]	Error [%]
1	0.480	52.000	2.462	146.200	0.999	0.100	1.002	0.200	1.020	2.000
0.82	0.330	59.756	0.377	54.024	0.819	0.122	0.825	0.610	0.800	2.439
0.73	0.270	63.014	0.769	5.342	0.729	0.137	0.733	0.411	0.760	4.110
0.64	0.170	73.438	2.061	222.031	0.639	0.156	0.646	0.937	0.660	3.125
0.55	0.140	74.545	1.125	104.545	0.549	0.182	0.557	1.273	0.570	3.636
0.46	0.110	76.087	0.691	50.217	0.459	0.217	0.466	1.304	0.470	2.174

The most accurate results for detection of the damping and frequencies were obtained by using the Ibrahim Time Domain method with errors in both of the order of 0.1%. The Rational Fraction Polynomial method was only slightly inferior. The Hilbert Envelope method also gave good results for the damping (frequency values were simply read off from peaks in the frequency domain). The Complex Exponential and Cepstrum methods showed quite accurate results for the frequencies, but not for the damping.

Table 3 shows the results for values of frequency and damping of the experimental measurements. The true results are of course not known. If on the basis of the simulated results, the ITDM is taken to be best, the RFPM and HEM seem to give reliable results (the latter for damping only). The CM and CEM give reasonable results for frequencies but poorer damping estimates. The damping results from the complex cepstrum are consistently lower than the others (as in Table 2). Since the complex Cepstrum comprises complex exponential terms further damped by a $1/n$ hyperbolic term, it is possible that the “differential cepstrum” which does not have this further damping [5], may provide better results. This will be tested in the future.

TABLE 3 - RESULTS OF THE EXPERIMENTAL DATA, POLES AND DAMPING

Freq.* [Hz]	CM		CEM		ITM		RFPM		HEM	
	Freq. [Hz]	Damp.** [%]	Freq. [Hz]	Damp. [%]	Freq. [Hz]	Damp. [%]	Freq. [Hz]	Damp. [%]	Freq. [Hz]	Damp. [%]
32	32.112	3.093	30.068	4.988	32.226	6.820	33.602	6.530	32	7.092
88	89.694	1.111	92.971	3.074	88.715	2.310	89.674	2.340	88	2.313
174	173.927	0.574	175.595	0.466	173.901	1.170	174.849	1.180	174	1.186
289	288.342	0.346	288.760	0.248	287.402	0.730	288.428	0.727	289	0.738
315	314.005	0.318	300.154	0.511	313.718	0.680	314.703	0.673	315	0.663
429	429.233	0.232	425.677	0.847	429.129	0.500	430.119	0.502	429	0.499
598	599.535	0.166	592.154	0.938	599.567	0.400	600.556	0.404	598	0.363
629	628.913	0.158	625.304	0.575	628.928	0.360	629.928	0.357	629	0.378
798	797.923	0.125	797.669	0.347	797.986	0.280	798.989	0.283	798	0.284
949	947.429	0.105	947.669	0.205	947.593	0.240	948.582	0.241	949	0.240

- The HEM analysed the envelope only and that is why only the damping was estimated.

*Frequency; ** Damping

5. CONCLUSIONS

The most accurate results for detection of the damping and frequency values were obtained by using the Ibrahim Time Domain method, with the Rational Fraction Polynomial method very similar, and the Hilbert Envelope method giving comparable damping estimates. The Cepstrum and Complex Exponential methods also gave reasonably accurate results for natural frequencies; but not for the damping.

6. ACKNOWLEDGEMENT

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